- **Confidence Interval for Population Mean With Replacement (Z-Test)**

When Population Standard Deviation (σ) is known

$$P\left[\bar{X}-Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\le\mu\le\bar{X}+Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]=1-\alpha$$

Or

$$\bar{X}-Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\le\mu\le\bar{X}+Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

When Population Standard Deviation (σ) is unknown & n>30

$$P\left[\bar{X}-Z_{\alpha/2}\frac{S}{\sqrt{n}}\le\mu\le\bar{X}+Z_{\alpha/2}\frac{S}{\sqrt{n}}\right]=1-\alpha$$

Or

$$\bar{X}-Z_{\alpha/2}\frac{S}{\sqrt{n}}\le\mu\le\bar{X}+Z_{\alpha/2}\frac{S}{\sqrt{n}}$$

- **Confidence Interval for Population Mean With Out Replacement (Z-Test)**

When Population Standard Deviation (σ) is known

$$P\left[\bar{X}-Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}\le\mu\le\bar{X}+Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}\right]=1-\alpha$$

Or

$$\bar{X}-Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}\le\mu\le\bar{X}+Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

When Population Standard Deviation (σ) is unknown

$$P\left[\bar{X}-Z_{\alpha/2}\frac{S}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}\le\mu\le\bar{X}+Z_{\alpha/2}\frac{S}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}\right]=1-\alpha$$

Or

$$\bar{X}-Z_{\alpha/2}\frac{S}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}\le\mu\le\bar{X}+Z_{\alpha/2}\frac{S}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

- **Confidence Interval for Difference Between Population Mean ($\mu_1-\mu_2$) With Replacement (Z-Test)**

When population S.D (σ) is known

$$P\left[\left(\bar{X}_1-\bar{X}_2\right)-Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\le\mu_1-\mu_2\le\left(\bar{X}_1-\bar{X}_2\right)+Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\right]=1-\alpha$$

Or

$$\left(\bar{X}_1-\bar{X}_2\right)-Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\le\mu_1-\mu_2\le\left(\bar{X}_1-\bar{X}_2\right)+Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

When population S.D (σ) is unknown & n_1, n_2 >30

$$P\left[\left(\bar{X}_1-\bar{X}_2\right)-Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\le\mu_1-\mu_2\le\left(\bar{X}_1-\bar{X}_2\right)+Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\right]=1-\alpha$$

Or

$$\left(\bar{X}_1-\bar{X}_2\right)-Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\le\mu_1-\mu_2\le\left(\bar{X}_1-\bar{X}_2\right)+Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}$$

- **Confidence Interval for Difference Between Population Mean ($\mu_2-\mu_1$) With Replacement (Z-Test)**

When population S.D (σ) is known

$$P\left[\left(\bar{X}_2-\bar{X}_1\right)-Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\le\mu_2-\mu_1\le\left(\bar{X}_2-\bar{X}_1\right)+Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\right]=1-\alpha$$

Or

$$\left(\bar{X}_2-\bar{X}_1\right)-Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\le\mu_2-\mu_1\le\left(\bar{X}_2-\bar{X}_1\right)+Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

When population S.D (σ) is unknown & n_1, n_2 >30

$$P\left[\left(\bar{X}_2-\bar{X}_1\right)-Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\le\mu_2-\mu_1\le\left(\bar{X}_2-\bar{X}_1\right)+Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\right]=1-\alpha$$

Or

$$\left(\bar{X}_2-\bar{X}_1\right)-Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\le\mu_2-\mu_1\le\left(\bar{X}_2-\bar{X}_1\right)+Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}$$

- **Confidence Interval for Difference Between Population Mean ($\mu_1-\mu_2$) With Out Replacement (Z-Test)**

When population S.D (σ) is known

$$P\left[\left(\bar{X}_1-\bar{X}_2\right)-Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{\sigma_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}\le\mu_1-\mu_2\le\left(\bar{X}_1-\bar{X}_2\right)+Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{\sigma_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}\right]=1-\alpha$$

Or

$$\left(\bar{X}_1-\bar{X}_2\right)-Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{\sigma_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}\le\mu_1-\mu_2\le\left(\bar{X}_1-\bar{X}_2\right)+Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{\sigma_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}$$

When population S.D (σ) is unknown & n_1, n_2 >30

$$P\left[\left(\bar{X}_1-\bar{X}_2\right)-Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{S_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}\le\mu_1-\mu_2\le\left(\bar{X}_1-\bar{X}_2\right)+Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{S_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}\right]=1-\alpha$$

Or

$$\left(\bar{X}_1-\bar{X}_2\right)-Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{S_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}\le\mu_1-\mu_2\le\left(\bar{X}_1-\bar{X}_2\right)+Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{S_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}$$

- **Confidence Interval for Difference Between Population Mean ($\mu_2-\mu_1$) With Out Replacement (Z-Test)**

When population S.D (σ) is known

$$P\left[\left(\bar{X}_2-\bar{X}_1\right)-Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{\sigma_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}\le\mu_2-\mu_1\le\left(\bar{X}_2-\bar{X}_1\right)+Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{\sigma_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}\right]=1-\alpha$$

Or

$$\left(\bar{X}_2-\bar{X}_1\right)-Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{\sigma_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}\le\mu_2-\mu_1\le\left(\bar{X}_2-\bar{X}_1\right)+Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{\sigma_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}$$

When population S.D (σ) is unknown & n_1, n_2 >30

$$P\left[\left(\bar{X}_2-\bar{X}_1\right)-Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{S_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}\le\mu_2-\mu_1\le\left(\bar{X}_2-\bar{X}_1\right)+Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{S_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}\right]=1-\alpha$$

Or

$$\left(\bar{X}_2-\bar{X}_1\right)-Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{S_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}\le\mu_2-\mu_1\le\left(\bar{X}_2-\bar{X}_1\right)+Z_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}\left(\frac{N_1-n_1}{N_1-1}\right)+\frac{S_2^2}{n_2}\left(\frac{N_2-n_2}{N_2-1}\right)}$$

- **Confidence Interval for Population Proportion (Z-Test)**

$$P\left[\hat{P}-Z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}\le P\le\hat{P}+Z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}\right]=1-\alpha$$

Or

$$\hat{P}-Z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}\le P\le\hat{P}+Z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$$

- **Confidence Interval for Difference Between Two Population Proportion ($P_1 - P_2$) (Z-Test)**

$$P\left[\left(\hat{P}_1-\hat{P}_2\right)-Z_{\alpha/2}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1}+\frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}\le P_1-P_2\le\left(\hat{P}_1-\hat{P}_2\right)+Z_{\alpha/2}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1}+\frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}\right]=1-\alpha$$

Or

$$\left(\hat{P}_1-\hat{P}_2\right)-Z_{\alpha/2}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1}+\frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}\le P_1-P_2\le\left(\hat{P}_1-\hat{P}_2\right)+Z_{\alpha/2}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1}+\frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}$$

- **Confidence Interval for Difference Between Two Population Proportion ($P_2 - P_1$) (Z-Test)**

$$P\left[\left(\hat{P}_2-\hat{P}_1\right)-Z_{\alpha/2}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1}+\frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}\le P_2-P_1\le\left(\hat{P}_2-\hat{P}_1\right)+Z_{\alpha/2}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1}+\frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}\right]=1-\alpha$$

Or

$$\left(\hat{P}_2-\hat{P}_1\right)-Z_{\alpha/2}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1}+\frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}\le P_2-P_1\le\left(\hat{P}_2-\hat{P}_1\right)+Z_{\alpha/2}\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1}+\frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}$$

- **Confidence Interval Estimate for Population Correlation Coefficient (Z-Test)**

$$P\left[Z_f-Z_{\alpha/2}\frac{1}{\sqrt{n-3}}\le\mu_z\le Z_f+Z_{\alpha/2}\frac{1}{\sqrt{n-3}}\right]=1-\alpha$$

Or

$$Z_f-Z_{\alpha/2}\frac{1}{\sqrt{n-3}}\le\mu_z\le Z_f+Z_{\alpha/2}\frac{1}{\sqrt{n-3}}$$

$$\therefore\ Z_f=1.1513\log\left[\frac{1+r}{1-r}\right]$$