

## • Testing of Hypotheses concerning the Population Mean (Z-Test)

### 1. Null & Alternative Hypotheses

Null	$H_0;$	$\mu = \mu_0$	$\mu \leq \mu_0$	$\mu \geq \mu_0$
Alternative	$H_1;$	$\mu \neq \mu_0$	$\mu > \mu_0$	$\mu < \mu_0$

### 2. Significance Level

$$\alpha = 5\% / 1\% \quad \text{or} \quad 0.05 / 0.01$$

If significance level is not given then we take 5% by default.

### 3. Critical Region(C.R)

**For Two Tail Test**  $\left( \frac{1-\alpha}{2} = Z_{\alpha/2} \right)$

If Alternative  $H_1; \mu \neq \mu_0$

$$\text{C.R} = |Z| \geq Z_{\alpha/2} \quad \text{or} \quad -Z_{\alpha/2} < Z < Z_{\alpha/2}$$

**For One Tail Test**  $(0.5 - \alpha = Z_{\alpha})$

If Alternative  $H_1; \mu > \mu_0$

$$\text{C.R} = Z \geq Z_{\alpha}$$

**For One Tail Test**  $(0.5 - \alpha = Z_{\alpha})$

If Alternative  $H_1; \mu < \mu_0$

$$\text{C.R} = Z \leq -Z_{\alpha}$$

### 4. Test Statistics

**When population S.D ( $\sigma$ ) is known**

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

**When population S.D ( $\sigma$ ) is unknown &  $n > 30$**

$$Z = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

### 5. Conclusion

- If z-cal is greater than or equal to z-tab so rejected  $H_0$
- If z-cal is less than z-tab so accepted  $H_0$

• **Testing of Hypotheses concerning the difference between two Population Mean (  $\bar{X}_1 - \bar{X}_2$  ) (Z-Test)**

1. **Null & Alternative Hypotheses**

Null	$H_0;$	$\mu_1 - \mu_2 = \Delta_0$	$\mu_1 - \mu_2 \leq \Delta_0$	$\mu_1 - \mu_2 \geq \Delta_0$
Alternative	$H_1;$	$\mu_1 - \mu_2 \neq \Delta_0$	$\mu_1 - \mu_2 > \Delta_0$	$\mu_1 - \mu_2 < \Delta_0$

2. **Significance Level**

$\alpha = 5\% / 1\%$  or  $0.05 / 0.01$

If significance level is not given then we take 5% by default.

3. **Critical Region(C.R)**

**For Two Tail Test (  $\frac{1-\alpha}{2} = Z_{\alpha/2}$  )**

If Alternative  $H_1; \mu_1 - \mu_2 \neq \Delta_0$

C.R =  $|Z| \geq Z_{\alpha/2}$  or  $-Z_{\alpha/2} < Z < Z_{\alpha/2}$

**For One Tail Test (  $0.5 - \alpha = Z_\alpha$  )**

If Alternative  $H_1; \mu_1 - \mu_2 > \Delta_0$

C.R =  $Z \geq Z_\alpha$

**For One Tail Test (  $0.5 - \alpha = Z_\alpha$  )**

If Alternative  $H_1; \mu_1 - \mu_2 < \Delta_0$

C.R =  $Z \leq -Z_\alpha$

4. **Test Statistics**

$\therefore \Delta_0 = \mu_1 - \mu_2$

**When population S.D (  $\sigma$  ) is known**

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

**When population S.D (  $\sigma$  ) is unknown &  $n_1, n_2 > 30$**

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

If  $\Delta_0$  is not given in question then we take  $\Delta_0 = 0$ .

5. **Conclusion**

- If z-cal is greater than or equal to z-tab so rejected  $H_0$
- If z-cal is less than z-tab so accepted  $H_0$

• **Testing of Hypotheses concerning the difference between two Population Mean (  $\bar{X}_2 - \bar{X}_1$  ) (Z-Test)**

1. **Null & Alternative Hypotheses**

Null	$H_0;$	$\mu_2 - \mu_1 = \Delta_0$	$\mu_2 - \mu_1 \leq \Delta_0$	$\mu_2 - \mu_1 \geq \Delta_0$
Alternative	$H_1;$	$\mu_2 - \mu_1 \neq \Delta_0$	$\mu_2 - \mu_1 > \Delta_0$	$\mu_2 - \mu_1 < \Delta_0$

2. **Significance Level**

$\alpha = 5\% / 1\%$  or  $0.05 / 0.01$

If significance level is not given then we take 5% by default.

3. **Critical Region(C.R)**

**For Two Tail Test (  $\frac{1-\alpha}{2} = Z_{\alpha/2}$  )**

If Alternative  $H_1; \mu_2 - \mu_1 \neq \Delta_0$

C.R =  $|Z| \geq Z_{\alpha/2}$  or  $-Z_{\alpha/2} < Z < Z_{\alpha/2}$

**For One Tail Test (  $0.5 - \alpha = Z_{\alpha}$  )**

If Alternative  $H_1; \mu_2 - \mu_1 > \Delta_0$

C.R =  $Z \geq Z_{\alpha}$

**For One Tail Test (  $0.5 - \alpha = Z_{\alpha}$  )**

If Alternative  $H_1; \mu_2 - \mu_1 < \Delta_0$

C.R =  $Z \leq -Z_{\alpha}$

4. **Test Statistics**

$\therefore \Delta_0 = \mu_2 - \mu_1$

**When population S.D (  $\sigma$  ) is known**

$$Z = \frac{(\bar{X}_2 - \bar{X}_1) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

**When population S.D (  $\sigma$  ) is unknown &  $n_1, n_2 > 30$**

$$Z = \frac{(\bar{X}_2 - \bar{X}_1) - \Delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

If  $\mu_0$  is not given in question then we take  $\mu_0 = 0$ .

### 5. Conclusion

- If z-cal is greater than or equal to z-tab so rejected  $H_0$
- If z-cal is less than z-tab so accepted  $H_0$

## • Testing of Hypotheses concerning the Population Proportion (Z-Test)

### 1. Null & Alternative Hypotheses

Null	$H_0; P = P_0$	$P \leq P_0$	$P \geq P_0$
Alternative	$H_1; P \neq P_0$	$P > P_0$	$P < P_0$

### 2. Significance Level

$$\alpha = 5\% / 1\% \quad \text{or} \quad 0.05 / 0.01$$

If significance level is not given then we take 5% by default.

### 3. Critical Region(C.R)

**For Two Tail Test**  $\left( \frac{1-\alpha}{2} = Z_{\alpha/2} \right)$

If Alternative  $H_1; P \neq P_0$

$$\text{C.R} = |Z| \geq Z_{\alpha/2} \quad \text{or} \quad -Z_{\alpha/2} < Z < Z_{\alpha/2}$$

**For One Tail Test**  $(0.5 - \alpha = Z_\alpha)$

If Alternative  $H_1; P > P_0$

$$\text{C.R} = Z \geq Z_\alpha$$

**For One Tail Test**  $(0.5 - \alpha = Z_\alpha)$

If Alternative  $H_1; P < P_0$

$$\text{C.R} = Z \leq -Z_\alpha$$

### 4. Test Statistics

$$Z = \frac{\hat{P} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} \quad \therefore \hat{P} = X/n$$

Or

$$Z = \frac{\hat{P} - P_0}{\sqrt{\frac{P_0 q_0}{n}}} \quad \therefore \hat{q} = 1 - \hat{P}$$

## 5. Conclusion

- If z-cal is greater than or equal to z-tab so rejected  $H_0$
- If z-cal is less than z-tab so accepted  $H_0$

## • Testing of Hypotheses concerning the difference between two Population Proportion ( $P_1 - P_2$ ) (Z-Test)

### 1. Null & Alternative Hypotheses

Null	$H_0;$	$P_1 - P_2 = \Delta_0$	$P_1 - P_2 \leq \Delta_0$	$P_1 - P_2 \geq \Delta_0$
Alternative	$H_1;$	$P_1 - P_2 \neq \Delta_0$	$P_1 - P_2 > \Delta_0$	$P_1 - P_2 < \Delta_0$

### 2. Significance Level

$$\alpha = 5\% / 1\% \quad \text{or} \quad 0.05 / 0.01$$

If significance level is not given then we take 5% by default.

### 3. Critical Region (C.R)

**For Two Tail Test** ( $\frac{1-\alpha}{2} = Z_{\alpha/2}$ )

If Alternative  $H_1; P_1 - P_2 \neq \Delta_0$

$$\text{C.R} = |Z| \geq Z_{\alpha/2} \quad \text{or} \quad -Z_{\alpha/2} < Z < Z_{\alpha/2}$$

**For One Tail Test** ( $0.5 - \alpha = Z_{\alpha}$ )

If Alternative  $H_1; P_1 - P_2 > \Delta_0$

$$\text{C.R} = Z \geq Z_{\alpha}$$

**For One Tail Test** ( $0.5 - \alpha = Z_{\alpha}$ )

If Alternative  $H_1; P_1 - P_2 < \Delta_0$

$$\text{C.R} = Z \leq -Z_{\alpha}$$

### 4. Test Statistics

$$\therefore \Delta_0 = P_1 - P_2$$

$$Z = \frac{(\hat{P}_1 - \hat{P}_2) - \Delta_0}{\sqrt{\frac{\hat{P}_1 \hat{q}_1}{n_1} + \frac{\hat{P}_2 \hat{q}_2}{n_2}}} \quad \therefore \hat{P}_1 = \frac{X_1}{n_1} \quad \hat{P}_2 = \frac{X_2}{n_2}$$

**Or**

$$Z = \frac{(\hat{P}_1 - \hat{P}_2) - \Delta_0}{\sqrt{\hat{P}_c \hat{q}_c \left[ \frac{1}{n_1} + \frac{1}{n_2} \right]}} \quad \therefore \hat{P}_c = \frac{n_1 \hat{P}_1 + n_2 \hat{P}_2}{n_1 + n_2} \quad \hat{q}_c = 1 - \hat{P}_c$$

If  $\Delta_0$  is not given in question then we take  $\Delta_0 = 0$ .

### 5. Conclusion

- If z-cal is greater than or equal to z-tab so rejected  $H_0$
- If z-cal is less than z-tab so accepted  $H_0$

### • Testing of Hypotheses concerning the difference between two Population Proportion ( $P_2 - P_1$ ) (Z-Test)

#### 1. Null & Alternative Hypotheses

Null	$H_0; \quad P_2 - P_1 = \Delta_0$	$P_2 - P_1 \leq \Delta_0$	$P_2 - P_1 \geq \Delta_0$
Alternative	$H_1; \quad P_2 - P_1 \neq \Delta_0$	$P_2 - P_1 > \Delta_0$	$P_2 - P_1 < \Delta_0$

#### 2. Significance Level

$$\alpha = 5\% / 1\% \quad \text{or} \quad 0.05 / 0.01$$

If significance level is not given then we take 5% by default.

#### 3. Critical Region (C.R)

**For Two Tail Test**  $\left( \frac{1-\alpha}{2} = Z_{\alpha/2} \right)$

If Alternative  $H_1; P_2 - P_1 \neq \Delta_0$

$$\text{C.R} = |Z| \geq Z_{\alpha/2} \quad \text{or} \quad -Z_{\alpha/2} < Z < Z_{\alpha/2}$$

**For One Tail Test**  $(0.5 - \alpha = Z_\alpha)$

If Alternative  $H_1; P_2 - P_1 > \Delta_0$

$$\text{C.R} = Z \geq Z_\alpha$$

**For One Tail Test**  $(0.5 - \alpha = Z_\alpha)$

If Alternative  $H_1; P_2 - P_1 < \Delta_0$

$$\text{C.R} = Z \leq -Z_\alpha$$

#### 4. Test Statistics

$$\therefore \Delta_0 = P_2 - P_1$$

$$Z = \frac{(\hat{P}_2 - \hat{P}_1) - \Delta_0}{\sqrt{\frac{\hat{P}_1 \hat{q}_1}{n_1} + \frac{\hat{P}_2 \hat{q}_2}{n_2}}}$$

$$\therefore \hat{P}_1 = X_1 / n_1$$

$$\hat{P}_2 = X_2 / n_2$$

Or

$$Z = \frac{(\hat{P}_2 - \hat{P}_1) - \Delta_0}{\sqrt{\hat{P}_c \hat{q}_c \left[ \frac{1}{n_1} + \frac{1}{n_2} \right]}}$$

$$\therefore \hat{P}_c = \frac{n_1 \hat{P}_1 + n_2 \hat{P}_2}{n_1 + n_2}$$

$$\hat{q}_c = 1 - \hat{P}_c$$

If  $\Delta_0$  is not given in question then we take  $\Delta_0 = 0$ .

#### 5. Conclusion

- If z-cal is greater than or equal to z-tab so rejected  $H_0$
- If z-cal is less than z-tab so accepted  $H_0$

#### • Testing of Hypotheses concerning the Population Correlation Coefficient when $(\rho = \rho_0 \text{ or } \rho \neq 0)$ (Z-Test) $\rho = \text{Row}$

##### 1. Null & Alternative Hypotheses

Null	$H_0;$	$\rho = \rho_0$	$\rho \leq \rho_0$	$\rho \geq \rho_0$
Alternative	$H_1;$	$\rho \neq \rho_0$	$\rho > \rho_0$	$\rho < \rho_0$

##### 2. Significance Level

$$\alpha = 5\% / 1\% \quad \text{or} \quad 0.05 / 0.01$$

If significance level is not given then we take 5% by default.

##### 3. Critical Region(C.R)

**For Two Tail Test**  $\left( \frac{1-\alpha}{2} = Z_{\alpha/2} \right)$

If Alternative  $H_1; \rho \neq \rho_0$

$$\text{C.R} = |Z| \geq Z_{\alpha/2} \quad \text{or} \quad -Z_{\alpha/2} < Z < Z_{\alpha/2}$$

**For One Tail Test**  $(0.5 - \alpha = Z_\alpha)$

If Alternative  $H_1; \rho > \rho_0$

$$\text{C.R} = Z \geq Z_\alpha$$

**For One Tail Test**  $(0.5 - \alpha = Z_\alpha)$

If Alternative  $H_1; \rho < \rho_0$

$$C.R = Z \leq -Z_{\alpha}$$

#### 4. Test Statistics

$$Z = \frac{Z_f - \mu_z}{\sigma_z}$$

$$Z_f = 1.1513 \log \left[ \frac{1+r}{1-r} \right]$$

$$\mu_z = 1.1513 \log \left[ \frac{1+\rho}{1-\rho} \right]$$

$$\sigma_z = \frac{1}{\sqrt{n-3}}$$

#### 5. Conclusion

- If z-cal is greater than or equal to z-tab so rejected  $H_0$
- If z-cal is less than z-tab so accepted  $H_0$

### • Testing of Hypotheses concerning the difference between Population of two Correlation Coefficient ( $r_1 - r_2$ ) (Z-Test) $\rho = Row$

#### 1. Null & Alternative Hypotheses

Null	$H_0; \rho_1 - \rho_2 = \Delta_0$	$\rho_1 - \rho_2 \leq \Delta_0$	$\rho_1 - \rho_2 \geq \Delta_0$
Alternative	$H_1; \rho_1 - \rho_2 \neq \Delta_0$	$\rho_1 - \rho_2 > \Delta_0$	$\rho_1 - \rho_2 < \Delta_0$

#### 2. Significance Level

$$\alpha = 5\% / 1\% \quad \text{or} \quad 0.05 / 0.01$$

If significance level is not given then we take 5% by default.

#### 3. Critical Region(C.R)

**For Two Tail Test** ( $\frac{1-\alpha}{2} = Z_{\alpha/2}$ )

If Alternative  $H_1; \rho_1 - \rho_2 \neq \Delta_0$

$$C.R = |Z| \geq Z_{\alpha/2} \quad \text{or} \quad -Z_{\alpha/2} < Z < Z_{\alpha/2}$$

**For One Tail Test** ( $0.5 - \alpha = Z_{\alpha}$ )

If Alternative  $H_1; \rho_1 - \rho_2 > \Delta_0$

$$C.R = Z \geq Z_{\alpha}$$

**For One Tail Test** ( $0.5 - \alpha = Z_{\alpha}$ )

If Alternative  $H_1; \rho_1 - \rho_2 < \Delta_0$

$$C.R = Z \leq -Z_\alpha$$

#### 4. Test Statistics

$$Z = \frac{(Z_{f1} - Z_{f2}) - \Delta_0}{\sigma_{Z1-Z2}} \quad \therefore \Delta_0 = \rho_1 - \rho_2$$

$$Z_{f1} = 1.1513 \log \left[ \frac{1+r_1}{1-r_1} \right] \quad Z_{f2} = 1.1513 \log \left[ \frac{1+r_2}{1-r_2} \right]$$

$$\sigma_{Z1-Z2} = \sqrt{\frac{1}{n_1-3} + \frac{1}{n_2-3}}$$

If  $\Delta_0$  is not given in question then we take  $\Delta_0 = 0$ .

#### 5. Conclusion

- If z-cal is greater than or equal to z-tab so rejected  $H_0$
- If z-cal is less than z-tab so accepted  $H_0$

### Testing of Hypotheses concerning the difference between Population of two Correlation Coefficient ( $r_2 - r_1$ ) (Z-Test) $\rho = Row$

#### 1. Null & Alternative Hypotheses

Null	$H_0; \rho_2 - \rho_1 = \Delta_0$	$\rho_2 - \rho_1 \leq \Delta_0$	$\rho_2 - \rho_1 \geq \Delta_0$
Alternative	$H_1; \rho_2 - \rho_1 \neq \Delta_0$	$\rho_2 - \rho_1 > \Delta_0$	$\rho_2 - \rho_1 < \Delta_0$

#### 2. Significance Level

$$\alpha = 5\% / 1\% \quad \text{or} \quad 0.05 / 0.01$$

If significance level is not given then we take 5% by default.

#### 3. Critical Region(C.R)

**For Two Tail Test** ( $\frac{1-\alpha}{2} = Z_{\alpha/2}$ )

If Alternative  $H_1; \rho_2 - \rho_1 \neq \Delta_0$

$$C.R = |Z| \geq Z_{\alpha/2} \quad \text{or} \quad -Z_{\alpha/2} < Z < Z_{\alpha/2}$$

**For One Tail Test** ( $0.5 - \alpha = Z_\alpha$ )

If Alternative  $H_1; \rho_2 - \rho_1 > \Delta_0$

$$C.R = Z \geq Z_\alpha$$

**For One Tail Test ( $0.5 - \alpha = Z_\alpha$ )**

If Alternative  $H_1; \rho_2 - \rho_1 < \Delta_0$

$$C.R = Z \leq -Z_\alpha$$

**4. Test Statistics**

$$Z = \frac{(Z_{f_2} - Z_{f_1}) - \Delta_0}{\sigma_{Z_2 - Z_1}} \quad \therefore \Delta_0 = \rho_2 - \rho_1$$

$$Z_{f_1} = 1.1513 \log \left[ \frac{1+r_1}{1-r_1} \right] \quad Z_{f_2} = 1.1513 \log \left[ \frac{1+r_2}{1-r_2} \right]$$

$$\sigma_{Z_2 - Z_1} = \sqrt{\frac{1}{n_1 - 3} + \frac{1}{n_2 - 3}}$$

If  $\Delta_0$  is not given in question then we take  $\Delta_0 = 0$ .

**5. Conclusion**

- If z-cal is greater than or equal to z-tab so rejected  $H_0$
- If z-cal is less than z-tab so accepted  $H_0$